Extending the Basic Local Independence Model for the assessment of (un)learning items in Knowledge Space Theory

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Theoretical Background

The Basic Local Independence Model (BLIM) has received much attention in probabilistic knowledge space theory (PKST):

$$P(R \mid K) = \left(\prod_{q \in K \setminus R} \beta_q\right) \cdot \left(\prod_{q \in K \cap R} (1-\beta_q)\right) \cdot \left(\prod_{q \in R \setminus K} \eta_q\right) \cdot \left(\prod_{q \in \overline{R} \cap \overline{K}} (1-\eta_q)\right)$$

In this work, we extend the BLIM for the longitudinal assessment of learning, i.e. estimating the probability of gaining or losing mastery of items between two points of measurement. Previous approaches have been formulated using an extension of PKST which involves sets of skills that are required to solve sets of items (Anselmi, Stefanutti, de Chiusole, & Robusto, 2017; Stefanutti, Anselmi, & Robusto, 2011).

Model Definitions

A BLIM for two points of measurement, assuming local independence of $P(R_t \mid K_t)$, reads:

$$P(R_1, R_2) = \sum_{K_1 \in \mathcal{K}} \sum_{K_2 \in \mathcal{K}} P(R_1 \mid K_1) \cdot P(R_2 \mid K_2) \cdot P(K_2 \mid K_1) \cdot P(K_1)$$

$\mathcal{K}$ is a well-graded knowledge space. The outer fringe is: $K^{\mathcal{O}} = \{q \in Q \setminus K : K \cup \{q\} \in \mathcal{K}\}$

We introduce the Item Gain Model (IGM) that only allows for gaining mastery of items:

$$P(K_2 \mid K_1) = \left(\prod_{q \in K_2 \setminus K_1} \gamma_q\right) \cdot \left(\prod_{q \in K_2^{\mathcal{O}}} (1-\gamma_q)\right)$$

and the Item Gain Loss Model (IGLM) that also allows for losing it:

$$P(K_2 \mid K_1) = \left(\prod_{q \in K_2 \setminus K_1} \gamma_q\right) \cdot \left(\prod_{q \in K_2^{\mathcal{O}} \setminus K_1} (1-\gamma_q)\right) \cdot \left(\prod_{q \in K_1 \cap K_2^{\mathcal{O}}} \lambda_q\right) \cdot \left(\prod_{q \in K_1 \cap K_2} (1-\lambda_q)\right)$$

Simulating Identifiability and Parameter Recovery

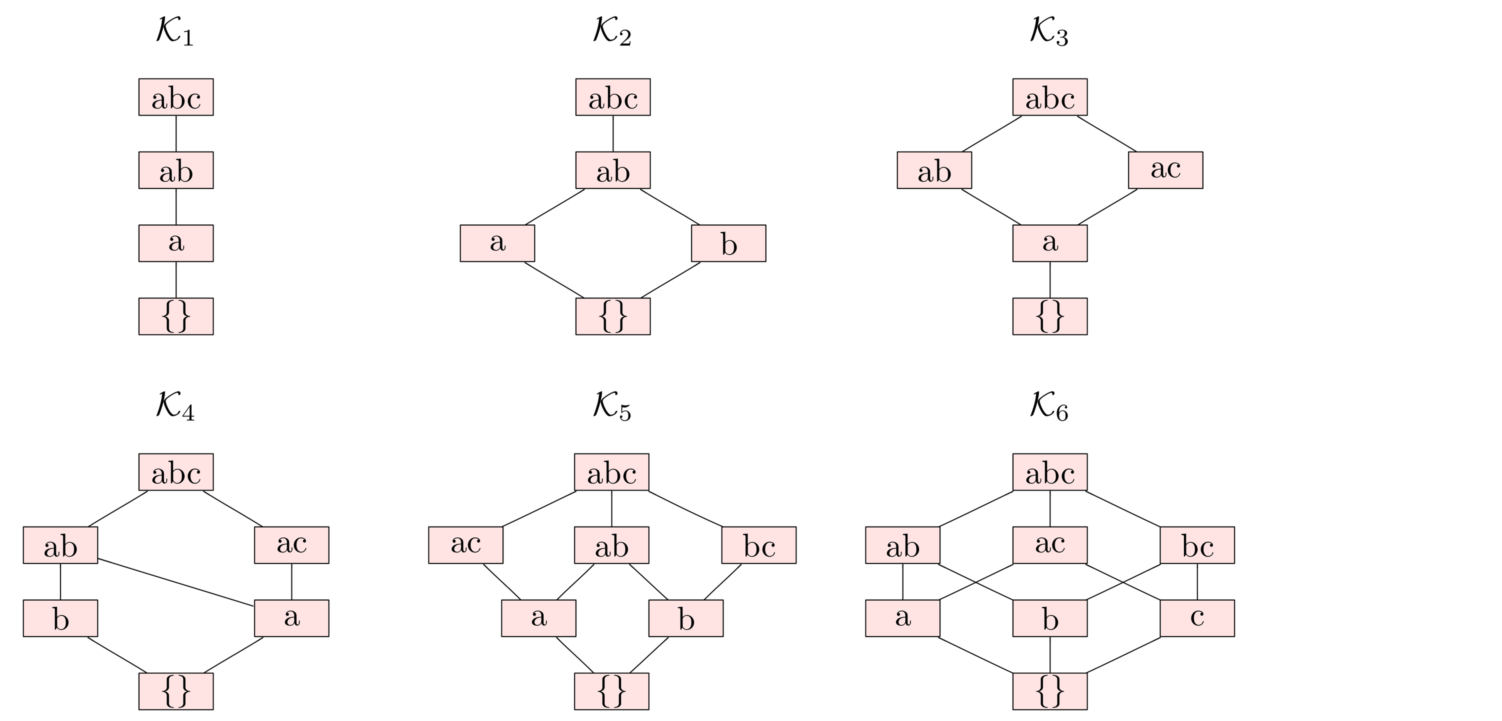
For $|Q| = 3$ there are six well-graded knowledge spaces, shown below. Model parameters were randomly drawn once from uniform distributions:

- $\eta_q, \beta_q \in [.03, .05]$ or $\eta_q, \beta_q \in [0, .01]$
- $\gamma_q \in [.4, .6]$ and for the IGLM: $\lambda_q \in [.1, .3]$
- $g_q \in [.4, .6]$ with $P(K_1) = \prod_{q \in K_1} g_q \prod_{q \in K_1^{\mathcal{O}}} (1-g_q)$

As has been established for the BLIM (Heller & Wickelmaier, 2013), parameters were estimated via a maximum likelihood expectation-maximization algorithm (ML) and a minimum discrepancy ML algorithm (MDML) (Maurer, 2020). In the latter, the likelihood is maximized subject to the constraint that pairs of states are only considered if each state is at a minimal distance $d(R, K) = |(R \setminus K) \cup (K \setminus R)|$ to its corresponding response pattern:

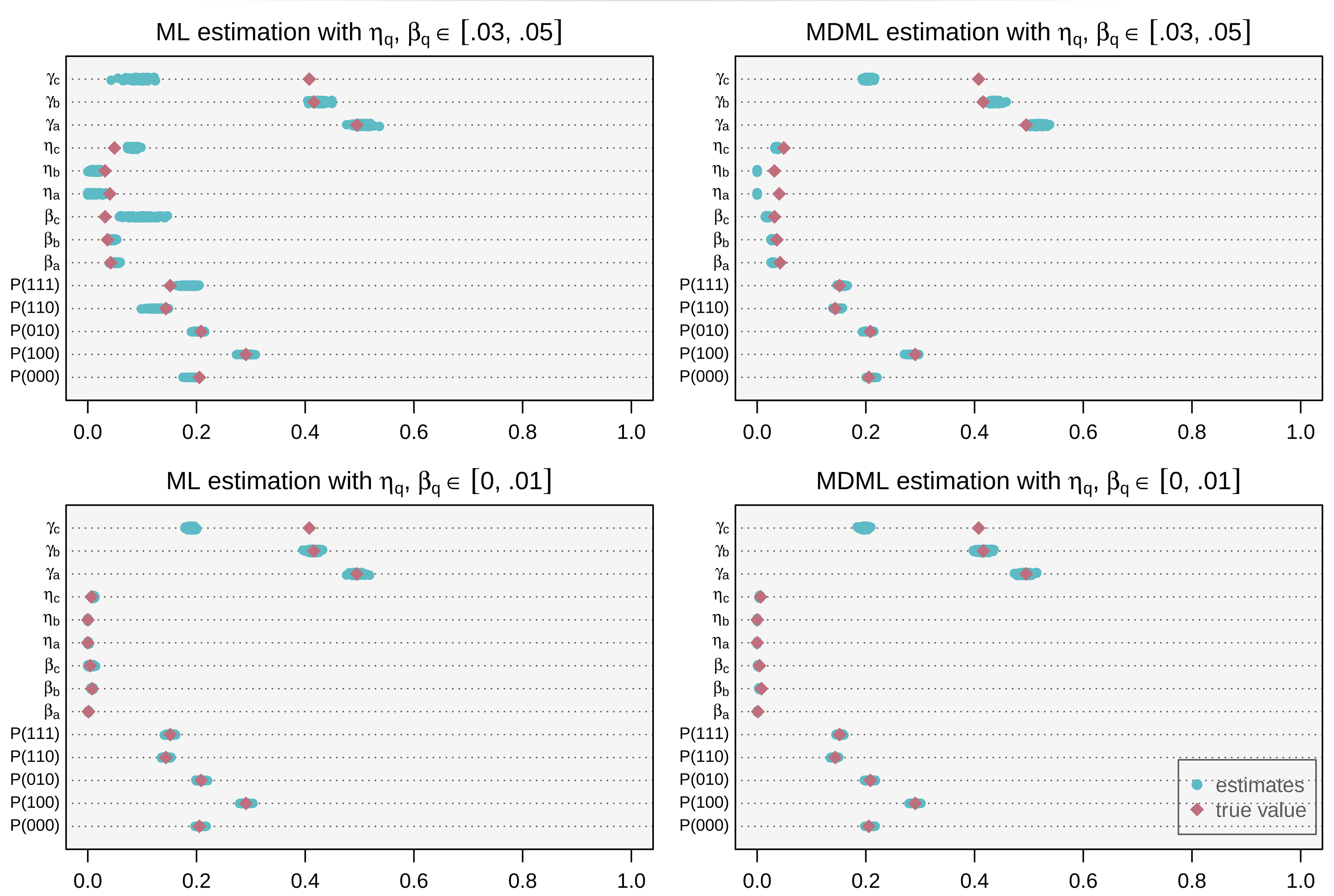
$$\min_{(K_1, K_2) \in \mathcal{K} \times \mathcal{K}} \{d(R_1, K_1) + d(R_2, K_2)\} = \min_{K \in \mathcal{K}} d(R_1, K) + \min_{K \in \mathcal{K}} d(R_2, K)$$

For each structure, we simulated 100 data sets of 10000 subjects each. Data were fitted using ML or MDML with a maximum of 20000 iterations.

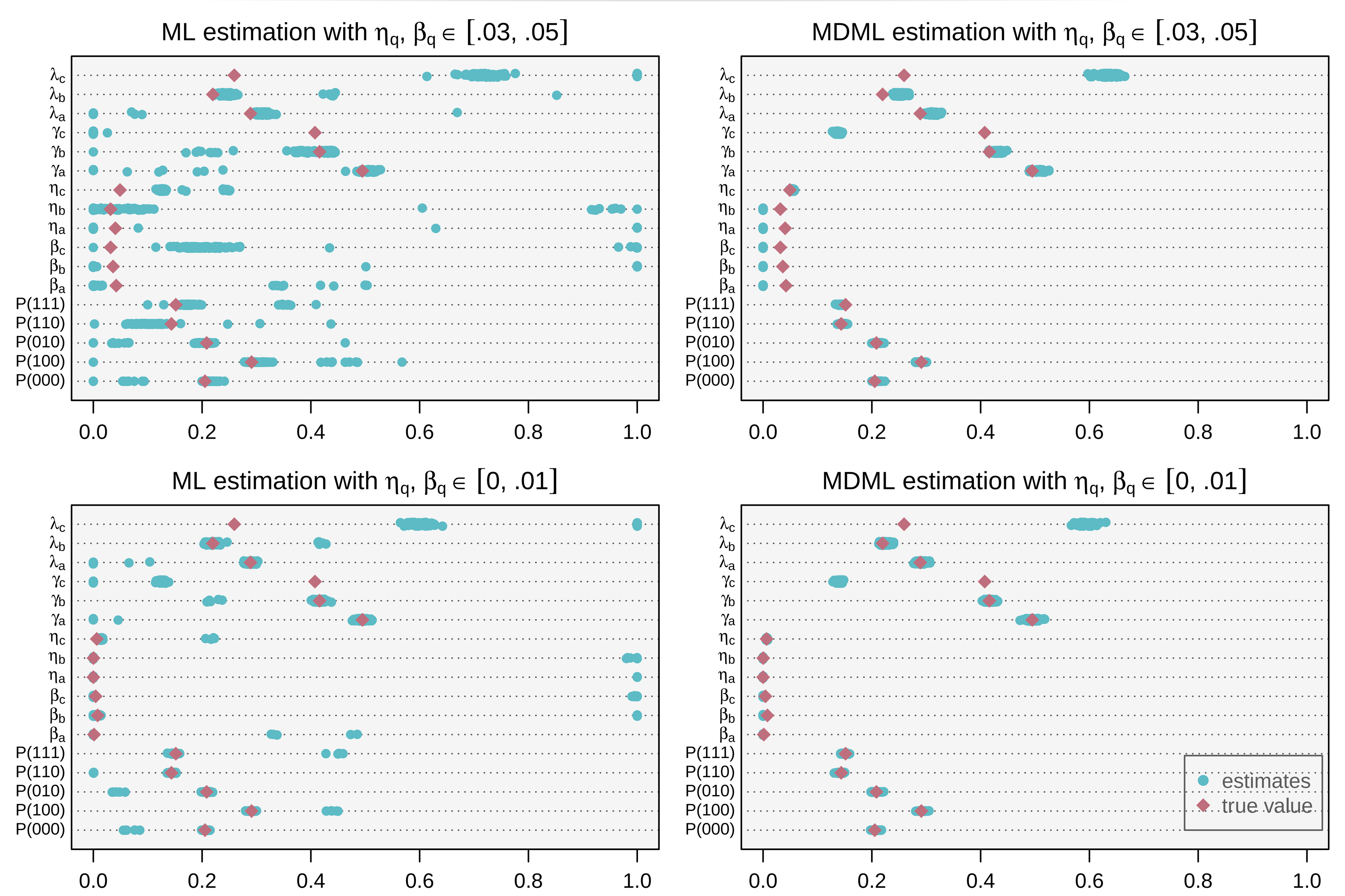


Results

IGM ($\mathcal{K}_2$)



IGLM ($\mathcal{K}_2$)



$\mathcal{K}$	Forw.-gr.			Back.-gr.			Bias (ML)			Bias (MDML)		
	a	b	c	a	b	c	γ_a	γ_b	γ_c	γ_a	γ_b	γ_c
$\mathcal{K}_1$	X					X		−.19	−.41		−.13	−.24
$\mathcal{K}_2$	X	X				X			−.31			−.20
$\mathcal{K}_3$	X				X	X		−.16	−.17		−.11	−.10
$\mathcal{K}_4$	X	X			X	X			−.17			−.10
$\mathcal{K}_5$	X	X				X			−.06			
$\mathcal{K}_6$	X	X	X	X	X	X						

Table 1: Bias ($|\text{median}(\hat{\theta}) - \theta| \geq .05$) for the IGM with $\eta_q, \beta_q \in [.03, .05]$

$\mathcal{K}$	Forw.-gr.			Back.-gr.			Bias (ML)						Bias (MDML)					
	a	b	c	a	b	c	γ_a	λ_a	γ_b	λ_b	γ_c	λ_c	γ_a	λ_a	γ_b	λ_b	γ_c	λ_c
$\mathcal{K}_1$	X					X			−.24	.27	−.41	.46			−.17	.25	−.29	.36
$\mathcal{K}_2$	X	X				X					−.41	.46					−.27	.37
$\mathcal{K}_3$	X				X	X			−.25	.29	−.27	.29			−.16	.25	−.15	.23
$\mathcal{K}_4$	X	X			X	X			−.17	−.22	−.28	.30					−.16	.25
$\mathcal{K}_5$	X	X				X					−.16	.14					−.06	.12
$\mathcal{K}_6$	X	X	X	X	X	X	−.29	−.20	−.18	−.22	−.13	−.19						

Table 2: Bias ($|\text{median}(\hat{\theta}) - \theta| \geq .05$) for the IGLM with $\eta_q, \beta_q \in [.03, .05]$

Conclusions

- Biases for γ_q, λ_q in both IGM and IGML ...
  - emerge for items in which $\mathcal{K}$ is not forward-graded
  - occur for both estimation methods and even for very small η_q and β_q
  - have the same sign for (almost) every fitted model: bias$(\gamma_q) < 0$ and bias$(\lambda_q) > 0$
  - are smaller (in absolute value) in more complex $\mathcal{K}$
- Parameter identifiability ...
  - is worse for IGLM compared to IGM
  - may be related to the complexity of $\mathcal{K}$
  - may be hard to separate from properties of the ML algorithm

Future Work

- Determine the causes for biases and (non-)identifiability
- Extend the models to incorporate different learning objects occuring between t_1 and t_2

References

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