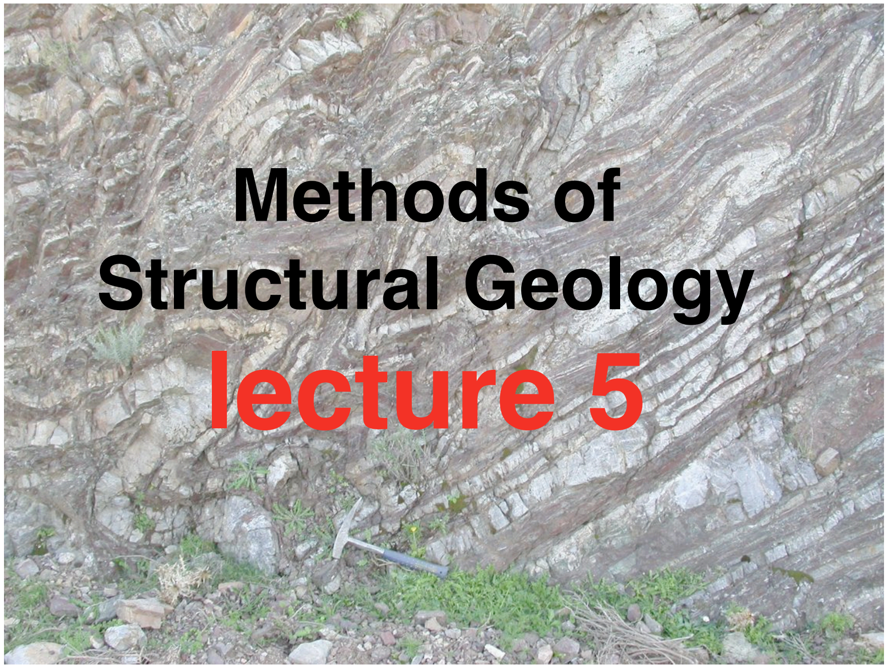
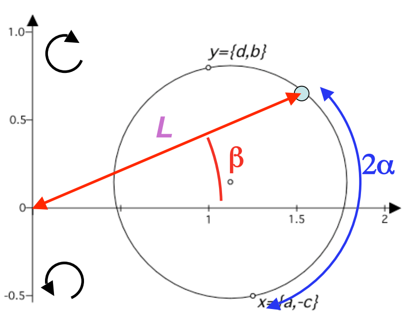


This lecture

- Last lecture
  - Introduction to the Mohr circle for strain
- This lecture
  - Short recap of last lecture
  - Discussion of exercise
  - The difference between pure and simple shear
  - Shortening and stretching

The Mohr circle for strain

- A line that makes an angle α with the X-axis
- Stretches by a factor L
- And rotates by an angle β



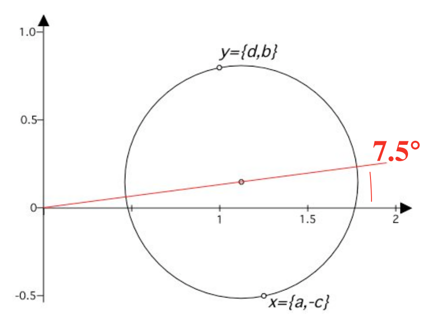
$$F_{ij} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The X-axis plots on **{a,-c}**

The Y-axis plots on **{d,b}**

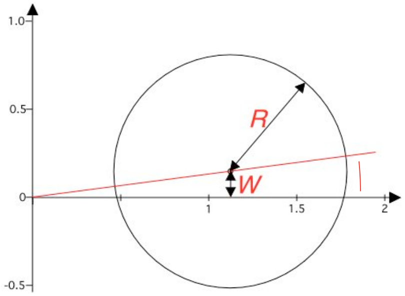
Vorticity

- Vorticity is the average rotation of lines
- Vorticity is strain dependent
  - Here: 7.5°



Kinematic vorticity number: W_k

- We need a "number" which tells us what the type of deformation is
- And that is independent of strain: W_k



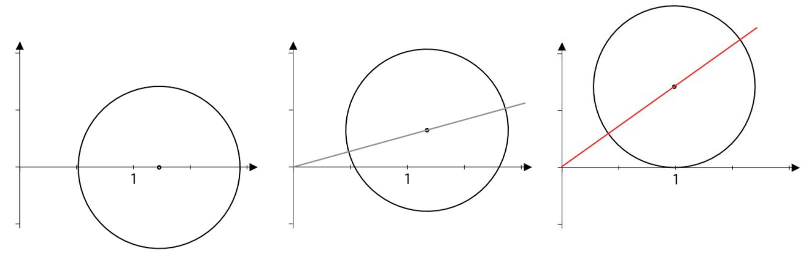
$$W_k = \frac{W}{R}$$

W_k and type of strain

- $W_k = 0$
- Pure shear

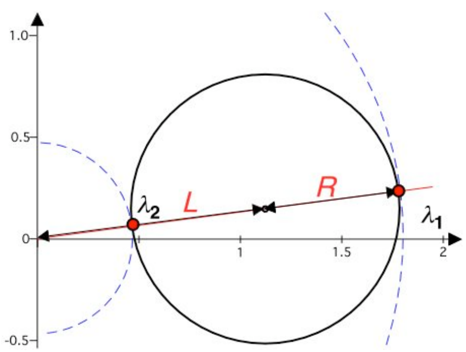
- $0 < W_k < 1$
- General shear

- $W_k = 1$
- Simple shear



Finite strain ratio and area change

- Maximum (λ_1) and minimum (λ_2) stretch are points on circle furthest and closest from origin



$$R_f = \frac{\lambda_1}{\lambda_2}$$

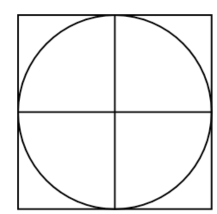
$$\Leftrightarrow R_f = \frac{L+R}{L-R}$$

Area change (ΔA)

$$\Delta A = \lambda_1 \lambda_2 - 1$$

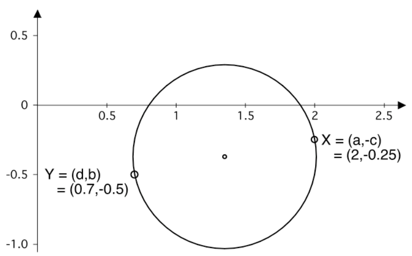
Exercise

- The position gradient tensor is: $F_{ij} = \begin{pmatrix} 2 & -0.5 \\ 0.25 & 0.7 \end{pmatrix}$
- Draw the Mohr circle
- How much do the X and Y axes stretch and rotate?
- What are R_f, ΔA, and W_k?
- What are the orientations of the finite stretching axes (FSAs) in the undeformed state?
- Draw the box shown here:
  - What does it look like in the deformed state?
  - In the undeformed and in the deformed state show the orientations that rotate to the left and to the right



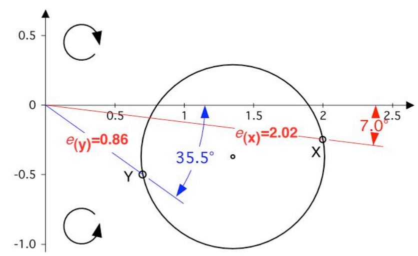
Exercise

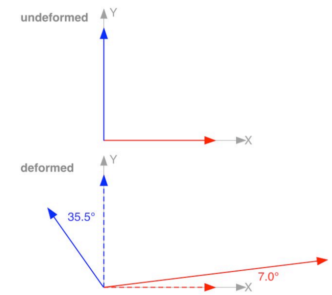
- The position gradient tensor is: $F_{ij} = \begin{pmatrix} 2 & -0.5 \\ 0.25 & 0.7 \end{pmatrix}$
- Draw the Mohr circle



Exercise

- The position gradient tensor is: $F_{ij} = \begin{pmatrix} 2 & -0.5 \\ 0.25 & 0.7 \end{pmatrix}$
- How much do the X and Y axes stretch and rotate?
  - **X-axis: $e_{(x)} = 2.02$, $\omega_{(x)} = 7.0°$**
  - **Y-axis: $e_{(y)} = 0.86$, $\omega_{(y)} = 35.5°$**





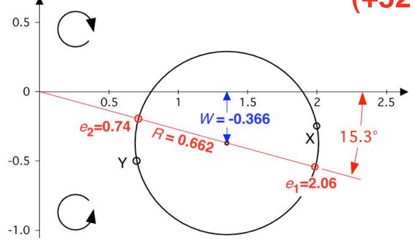
Exercise

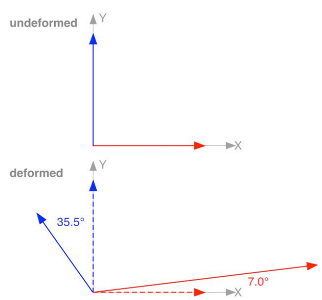
- The position gradient tensor is: $F_{ij} = \begin{pmatrix} 2 & -0.5 \\ 0.25 & 0.7 \end{pmatrix}$
- What are R_f, ΔA, and W_k?

R_f = 2.06/0.74 = 2.78

W_k = -0.366/0.662 = -0.55

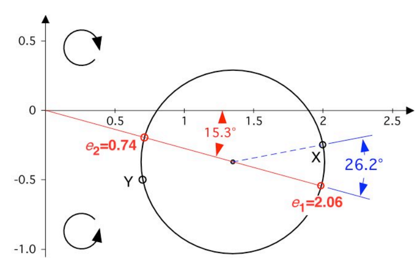
ΔA = 2.06·0.74-1 = 0.52 (+52%)

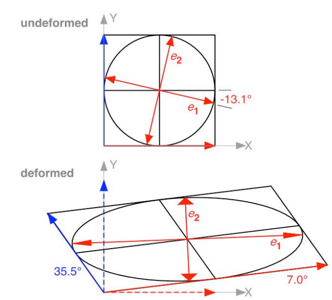




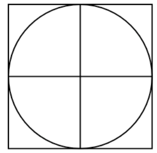
Exercise

- What are the orientations of the finite stretching axes (FSAs) in the undeformed state?
  - **e_1 makes angle of -26.2/2 = -13.1° with the X-axis**
  - **And rotated 15.3°**

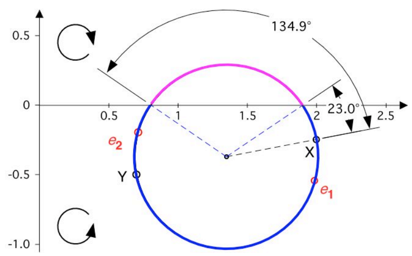


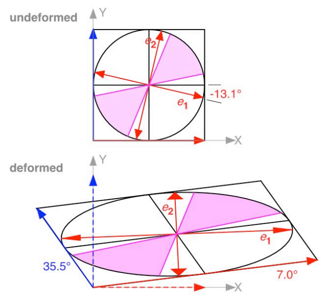


Exercise

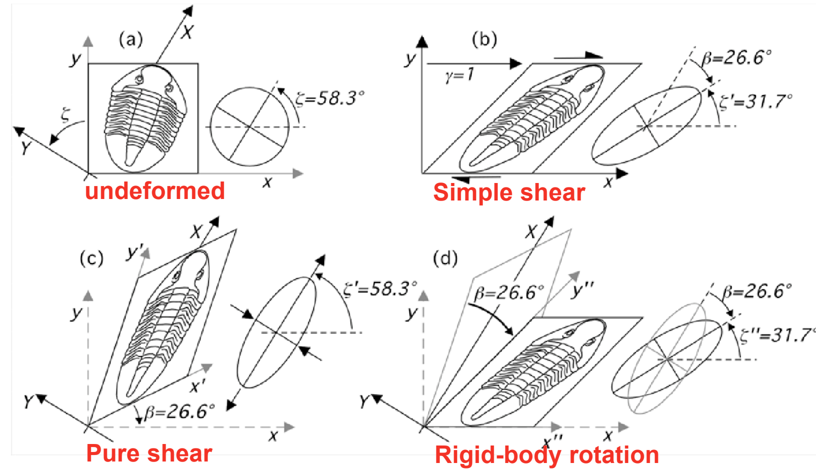


- Draw the box shown here:
  - What does it look like in the deformed state?
  - In the undeformed and in the deformed state show the orientations that rotate to the left and to the right

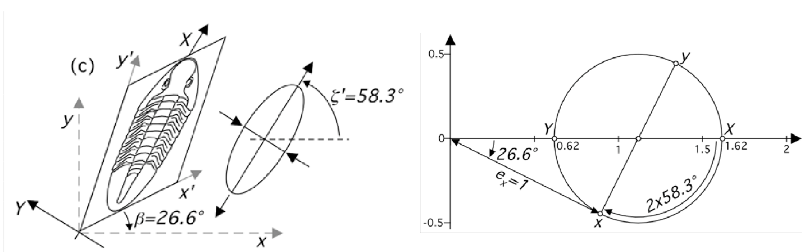




Different paths to same result



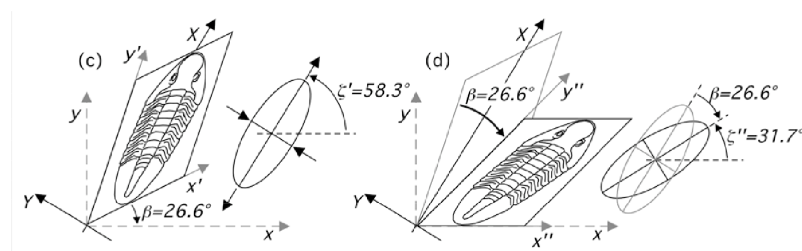
Pure shear

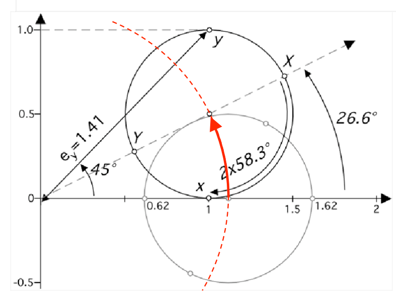


- Pure shear deformation with FSA's parallel to X-Y
  - $\lambda_1 = 1.62$
  - $\lambda_2 = 0.62$

  $R_f = 2.61$ $\Delta A = 0$ (no area change)
- X makes angle of 58.3° with x-axis
- The x-axes rotates 26.6°

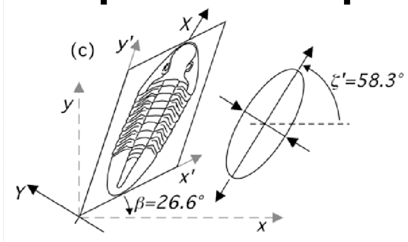
Pure shear + rotation

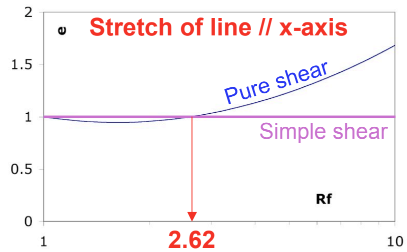




- Adding rigid-body rotation of 26.6° brings deformed x-axis back to horizontal (0° rotation)
- Rigid-body rotation means rotating all lines by same amount:
- Rotate Mohr circle around origin
- Result = simple shear (or is it?)

Simple shear $\stackrel{?}{=}$ pure shear + rotation

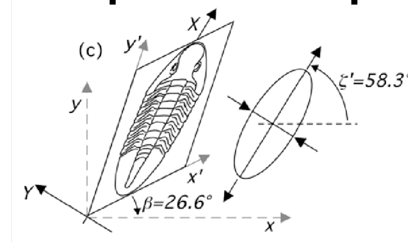


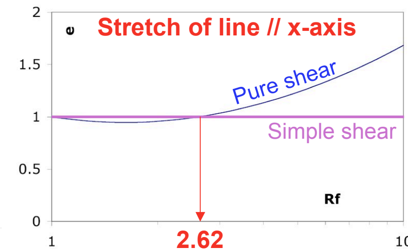


- With pure shear at 58.3° to the *x*-axis
- The line // x-axis first shortens and then stretches again
- At R_f = 2.62 it has a stretch of exactly $e_{(x)} = 1$

- With simple shear // *x*-axis
- The line // *x*-axis does not stretch or shorten ever

Simple shear ≠ pure shear + rotation

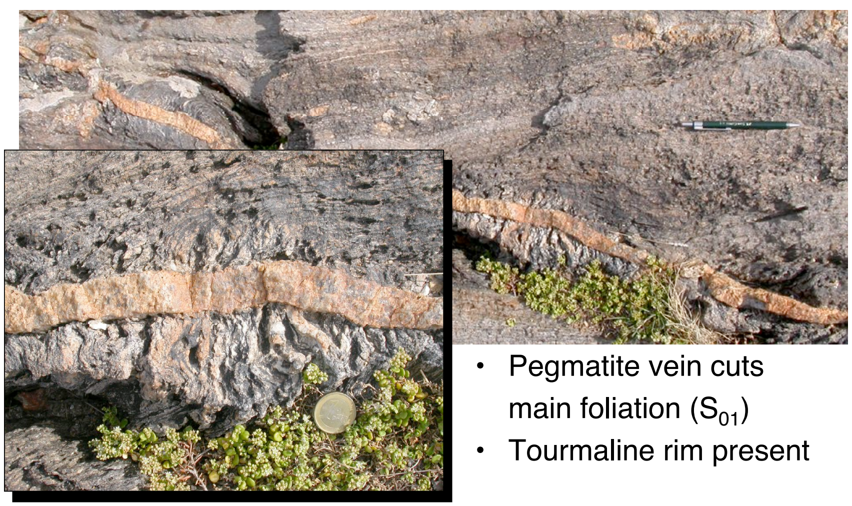




- With pure shear at 58.3° to the *x*-axis
- The line // x-axis first shortens and then stretches again
- At R_f = 2.62 it has a stretch of exactly $e_{(x)} = 1$

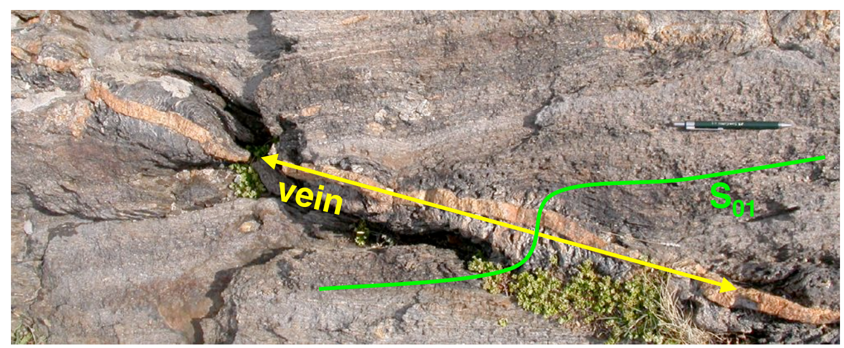
- With simple shear // *x*-axis
- The line // *x*-axis does not stretch or shorten ever

Pegmatite vein at Cap de Creus

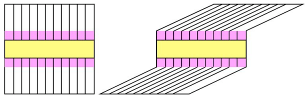


- Pegmatite vein cuts main foliation (S_{01})
- Tourmaline rim present

What are strain and kinematics?

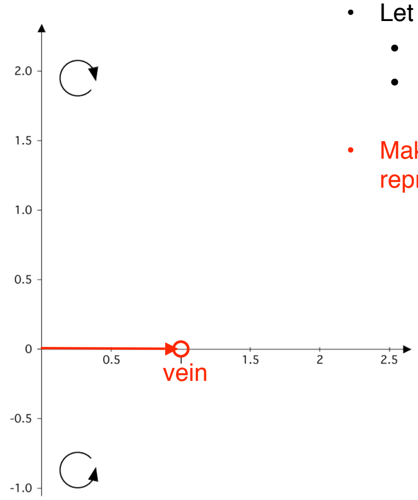


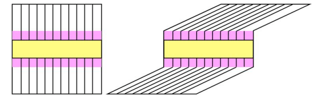
- Pegmatite vein no stretch: $e_{(v)} \approx 1.0$
- In tourmaline rim main foliation (S_{01}) originally at 90° to vein
- S_{01} at 25° to vein away from rim: rotated 65° clockwise



Simple shear // vein

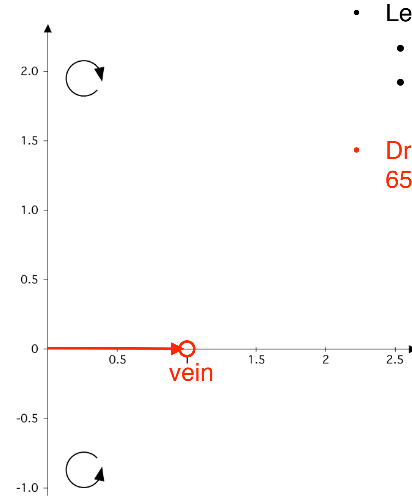
- Let us assume simple shear // vein
  - Vein does not stretch: $e_{(v)} = 1.0$
  - Vein does not rotate $\beta_{(v)} = 0°$
- Make a Mohr graph and plot the point representing the vein

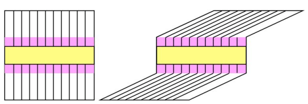




Simple shear // vein

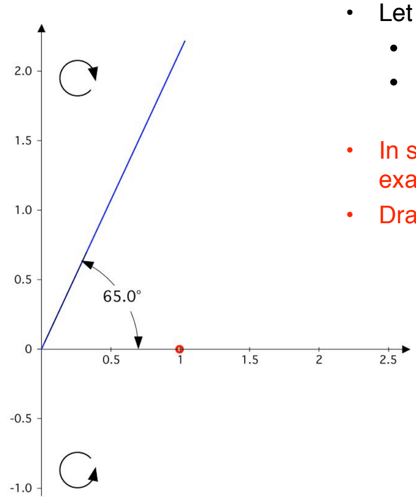
- Let us assume simple shear // vein
  - Vein does not stretch: $e_{(v)} = 1.0$
  - Vein does not rotate $\beta_{(v)} = 0°$
- Draw the line with all points that rotate 65°

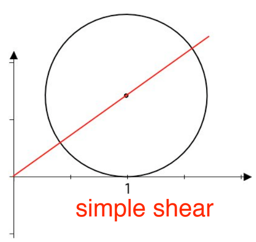




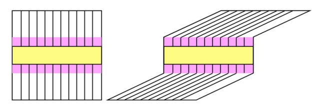
Simple shear // vein

- Let us assume simple shear // vein
  - Vein does not stretch: $e_{(v)} = 1.0$
  - Vein does not rotate $\beta_{(v)} = 0°$
- In simple shear, the Mohr circle lies exactly on the horizontal axis.
- Draw the Mohr circle

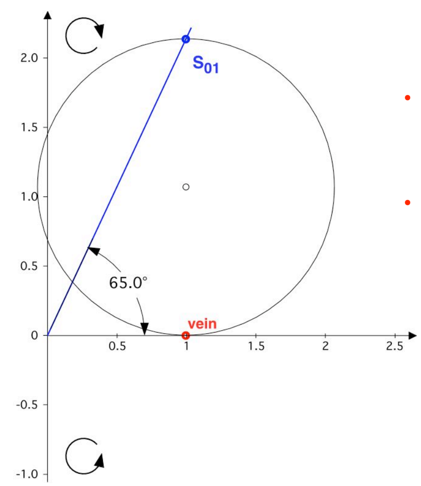




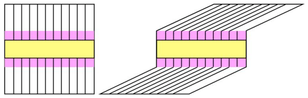
simple shear



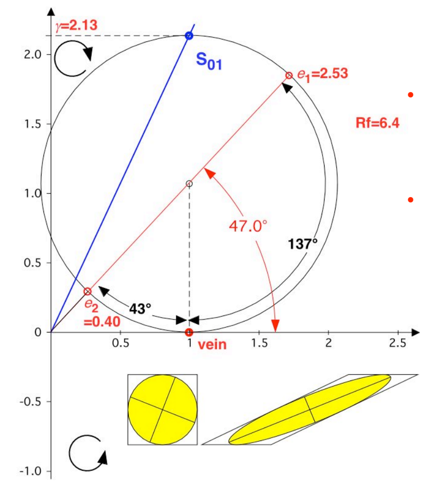
Simple shear // vein



- What are the shear strain and R_f?
- What is the position gradient tensor
- What are the stretches and rotations of the FSA's?
- What are the original and finite orientations of the FSA's relative to the vein?



Simple shear // vein

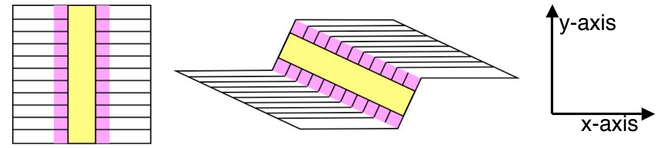


- What are the shear strain and R_f?
  - $\gamma = 2.13$
  - $R_f = 6.4$
- What is the position gradient tensor

$$F = \begin{pmatrix} 1 & 2.13 \\ 0 & 1 \end{pmatrix}$$

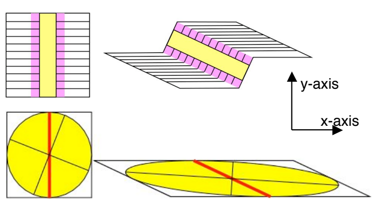
- What are the stretches and rotations of the FSA's?
  - $e_1 = 2.53$, $\beta = 47°$
  - $e_2 = 0.40$, $\beta = 47°$
- What are the original and finite orientations of the FSA's relative to the vein?
  - $e_1 = 68.5°$, $e_{1(\text{finite})} = 68.5\text{-}47\text{=}21.5°$
  - $e_2 = \text{-}21.5°$, $e_{2(\text{finite})} = \text{-}21.5\text{-}47\text{=-}68.5°$

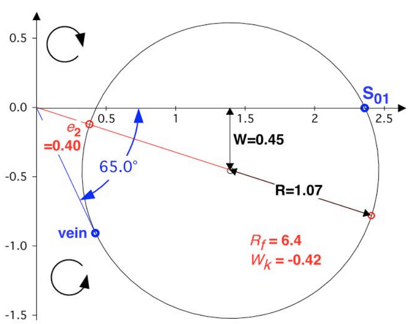
Case 2: foliation does not rotate



- Now suppose the foliation did not rotate
  - Define foliation // x-axis
- Draw the new Mohr circle
  - What is the vorticity number?
  - What is the position gradient tensor?
- Is this scenario likely?
  - Consider the stretch history of the vein
- (still assume no area change)

Case 2: foliation does not rotate





- Now suppose the foliation did not rotate
  - Define foliation // x-axis
- Draw the new Mohr circle
  - What is W_k? **0.45**
  - What is the position gradient tensor?

$$F = \begin{pmatrix} 2.35 & -0.91 \\ 0 & 0.42 \end{pmatrix}$$

- Is this scenario likely?
  - Consider the stretch history of the vein