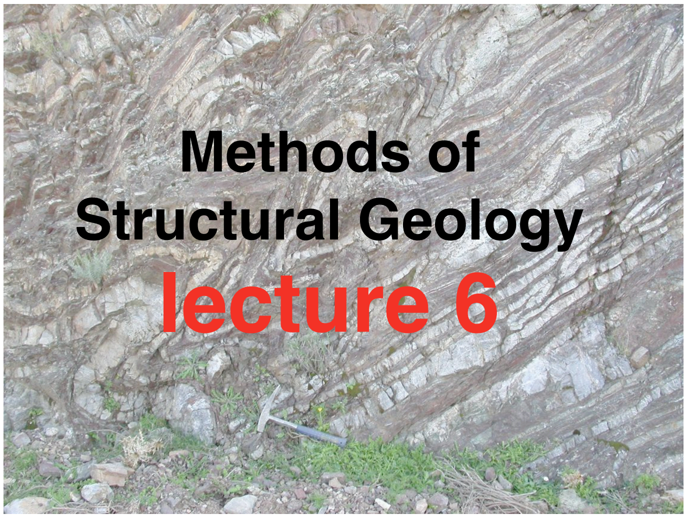
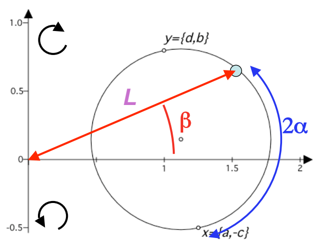


This lecture

- Last lectures
  - Mohr circle for strain
- This lecture
  - Look at deformation history of individual lines/planes
  - Different deformation histories: same result?
  - The difference between pure and simple shear
  - Stretching & shortening

The Mohr circle for strain

- A line that makes an angle α with the X-axis
- Stretches by a factor L
- And rotates by an angle β

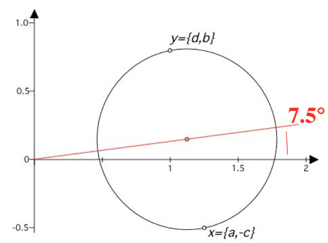


$$F_{ij} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The X-axis plots on **{a,-c}**
The Y-axis plots on **{d,b}**

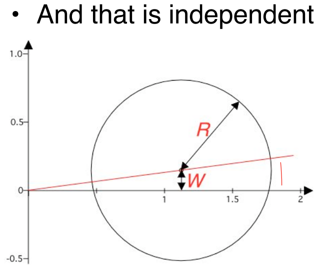
Vorticity

- Vorticity is the average rotation of lines
- Vorticity is strain dependent
  - Here: 7.5°



Kinematic vorticity number: W_k

- We need a "number" which tells us what the type of deformation is
- And that is independent of strain: W_k



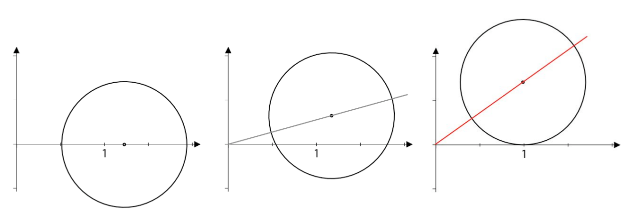
$$W_k = \frac{W}{R}$$

W_k and type of strain

- $W_k = 0$
- Pure shear

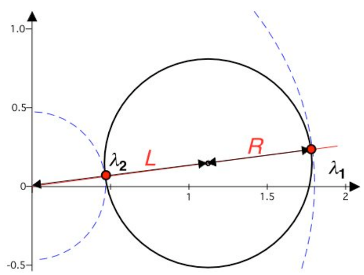
- $0 < W_k < 1$
- General shear

- $W_k = 1$
- Simple shear



Finite strain ratio and area change

- Maximum (λ_1) and minimum (λ_2) stretch are points on circle furthest and closest from origin



$$R_f = \frac{\lambda_1}{\lambda_2}$$

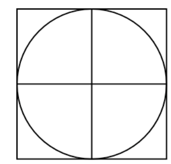
$$\Leftrightarrow R_f = \frac{L+R}{L-R}$$

Area change (ΔA)

$$\Delta A = \lambda_1 \lambda_2 - 1$$

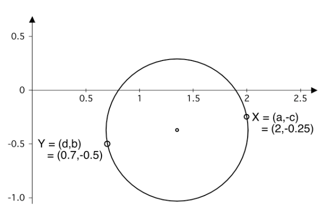
Exercise

- The position gradient tensor is: $F_{ij} = \begin{pmatrix} 2 & -0.5 \\ 0.25 & 0.7 \end{pmatrix}$
- Draw the Mohr circle
- How much do the X and Y axes stretch and rotate?
- What are R_f, ΔA, and W_k?
- What are the orientations of the finite stretching axes (FSAs) in the undeformed state?
- Draw the box shown here:
  - What does it look like in the deformed state?
  - In the undeformed and in the deformed state show the orientations that rotate to the left and to the right



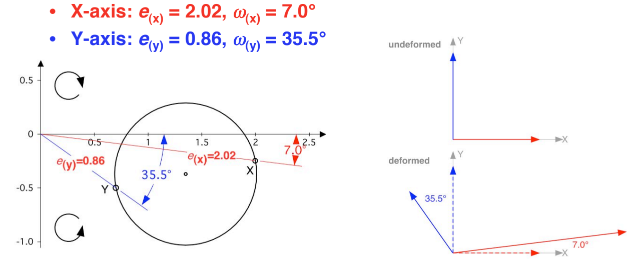
Exercise

- The position gradient tensor is: $F_{ij} = \begin{pmatrix} 2 & -0.5 \\ 0.25 & 0.7 \end{pmatrix}$
- Draw the Mohr circle



Exercise

- The position gradient tensor is: $F_{ij} = \begin{pmatrix} 2 & -0.5 \\ 0.25 & 0.7 \end{pmatrix}$
- How much do the X and Y axes stretch and rotate?
  - **X-axis: $e_{(x)} = 2.02$, $\omega_{(x)} = 7.0°$**
  - **Y-axis: $e_{(y)} = 0.86$, $\omega_{(y)} = 35.5°$**



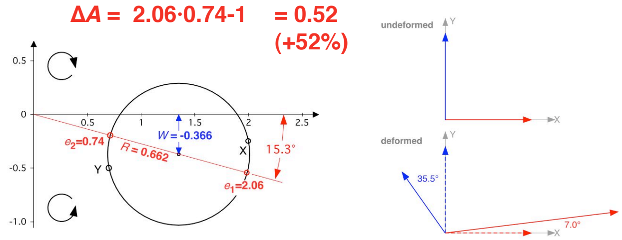
Exercise

- The position gradient tensor is: $F_{ij} = \begin{pmatrix} 2 & -0.5 \\ 0.25 & 0.7 \end{pmatrix}$
- What are R_f, ΔA, and W_k?

R_f = 2.06/0.74 = 2.78

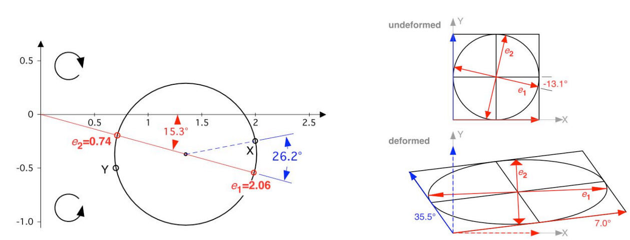
W_k = -0.366/0.662 = -0.55

ΔA = 2.06·0.74-1 = 0.52 (+52%)

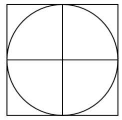


Exercise

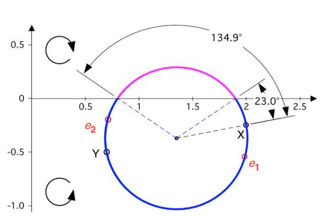
- What are the orientations of the finite stretching axes (FSAs) in the undeformed state?
  - **e_1 makes angle of -26.2/2 = -13.1° with the X-axis**
  - **And rotated 15.3°**

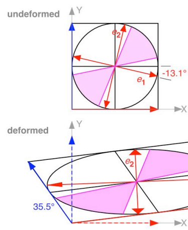


Exercise

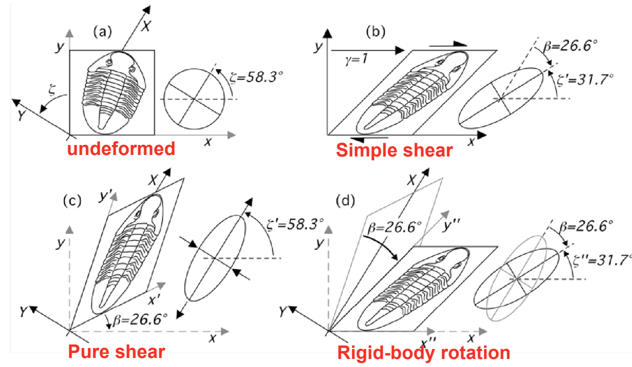


- Draw the box shown here:
  - What does it look like in the deformed state?
  - In the undeformed and in the deformed state show the orientations that rotate to the left and to the right

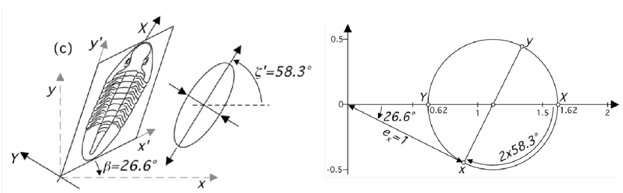




Different paths to same result



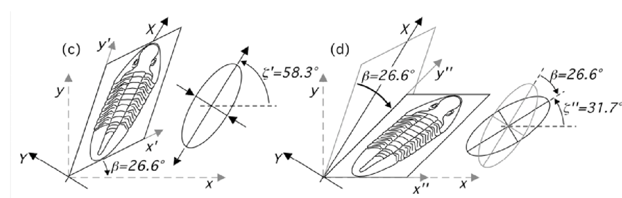
Pure shear

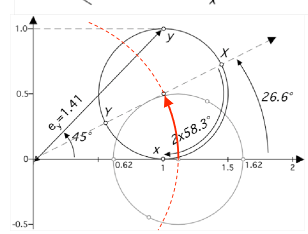


- Pure shear deformation with FSA's parallel to *X*-*Y*
  - λ_1 = 1.62
  - λ_2 = 0.62

  R_f = 2.61 ΔA= 0 (no area change)
- *X* makes angle of 58.3° with *x*-axis
- The x-axes rotates 26.6°

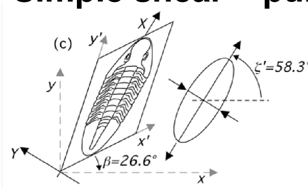
Pure shear + rotation

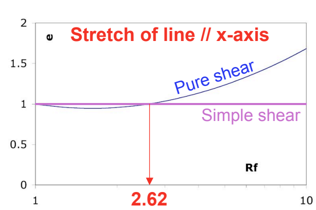




- Adding rigid-body rotation of 26.6° brings deformed x-axis back to horizontal (0° rotation)
- Rigid-body rotation means rotating all lines by same amount:
- Rotate Mohr circle around origin
- Result = simple shear (or is it?)

Simple shear ≟ pure shear + rotation

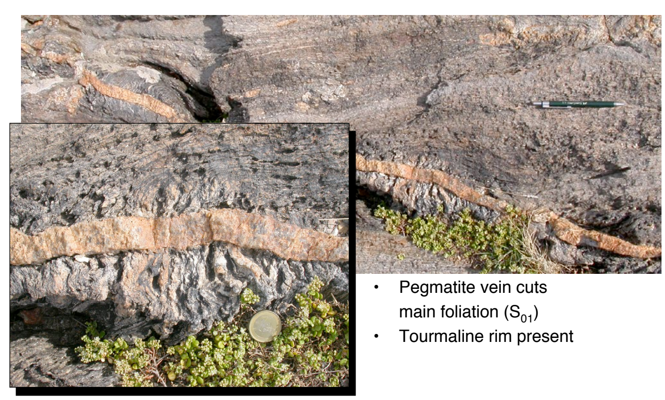




- **With pure shear at 58.3° to the *x*-axis**
- **The line // x-axis first shortens and then stretches again**
- **At R_f = 2.62 it has a stretch of exactly $e_{(x)}$ = 1**

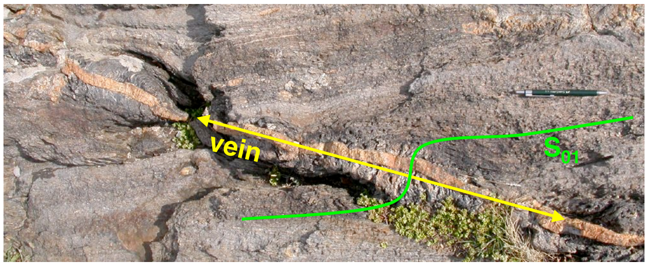
- **With simple shear // *x*-axis**
- **The line // *x*-axis does not stretch or shorten ever**

Pegmatite vein at Cap de Creus

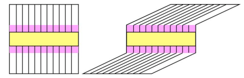


- Pegmatite vein cuts main foliation (S_{01})
- Tourmaline rim present

What are strain and kinematics?

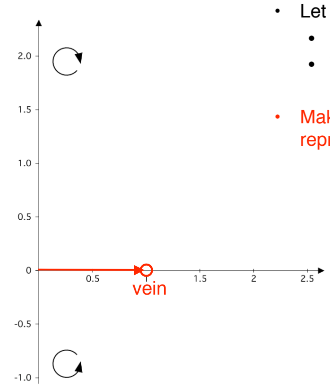


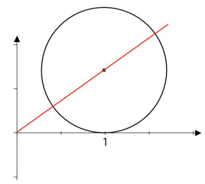
- Pegmatite vein no stretch: $e_{(v)} \approx 1.0$
- In tourmaline rim main foliation (S_{01}) originally at 90° to vein
- S_{01} at 25° to vein away from rim: rotated 65° clockwise



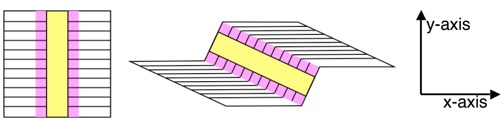
Simple shear // vein

- Let us assume simple shear // vein
  - Vein does not stretch: $e_{(v)} = 1.0$
  - Vein does not rotate $\beta_{(v)} = 0°$
- Make a Mohr graph and plot the point representing the vein





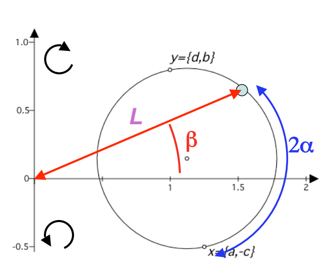
Case 2: foliation does not rotate



- Now suppose the foliation did not rotate
  - Define foliation // x-axis
- Draw the new Mohr circle
  - What is the vorticity number?
  - What is the position gradient tensor?
- Is this scenario likely?
  - Consider the stretch history of the vein
- (still assume no area change)

The Mohr circle for strain

- A line that makes an angle α with the X-axis
- Stretches by a factor L
- And rotates by an angle β



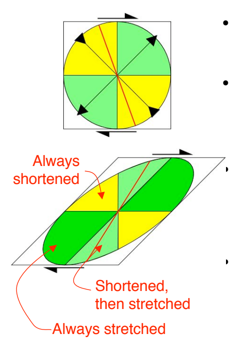
$$F_{ij} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The X-axis plots on **{a,-c}**
The Y-axis plots on **{d,b}**

Shortening, then stretching

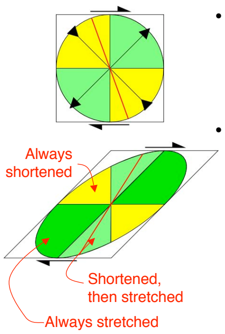
- Some layers may appear both stretched and shortened: folding AND boudinage

Instantaneous stretching & shortening: simple shear



- The instantaneous stretching axes (ISA's) are at 45° to the shear plane
- There a 4 quadrants
  - 2 of instantaneously shortening lines
  - 2 of instantaneously stretching lines
- The orientations of ISA's do not change during steady-state deformation
- Lines rotate and may rotate from a shortening field into a stretching field

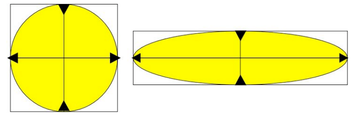
Instantaneous stretching & shortening: simple shear



- Draw two Mohr circles for progressive simple shear
  - One for γ=1
  - One for γ =2
- Determine for both finite shear strains the fields of
  - Lines that always shortened
  - Lines that shortened, then stretched
  - Lines that stretched, then shortened
  - Lines that always stretched
  - Lines that have a finite stretch of ***e***>1

Now for pure shear

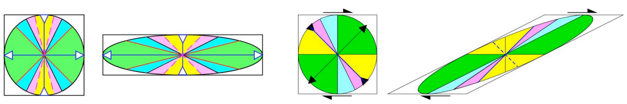
- Repeat the previous exercise for pure shear
- Use as the position gradient tensor:



$$F = \begin{pmatrix} 2 & 0 \\ 0 & 0.5 \end{pmatrix}$$

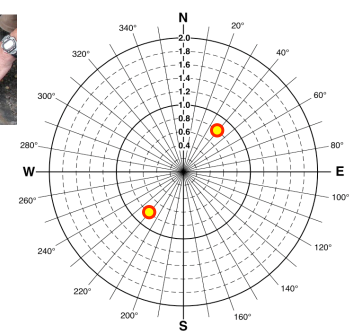
- Compare the result with that for simple shear
- Can you think of field observations that help to determine the kinematics of flow?

Pure *versus* simple shear



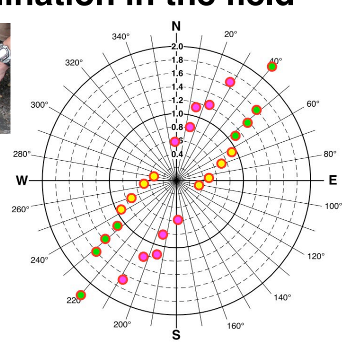
- Pure shear:
  - Shortened, then stretched lines distributed **symmetrically** around FSA
- Simple shear:
  - Shortened, then stretched lines distributed **asymmetrically** around FSA

Practical determination in the field



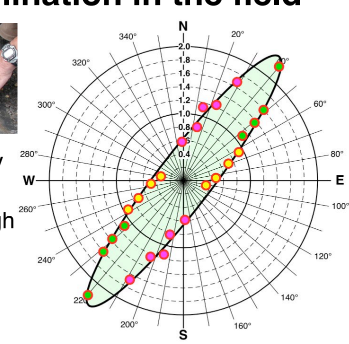
- Stretching/shortening can be measured in the field,
  - E.g. from veins
- Plot all measurements in stretch-direction plot
  - E.g. 0.8 in direction 040°
  - Equals 0.8 in direction 220°

Practical determination in the field



- Example of result for many measurements

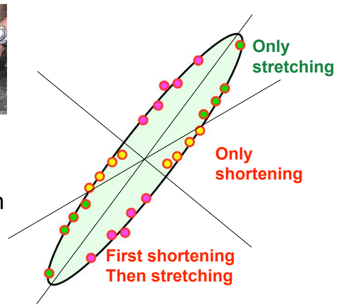
Practical determination in the field



- Example of result for many measurements
- Draw best-fit ellipse through data
  - $\boldsymbol{e}_1$ = 2.4
  - $\boldsymbol{e}_2$ = 0.4
  - $\boldsymbol{R}_f$ = 6

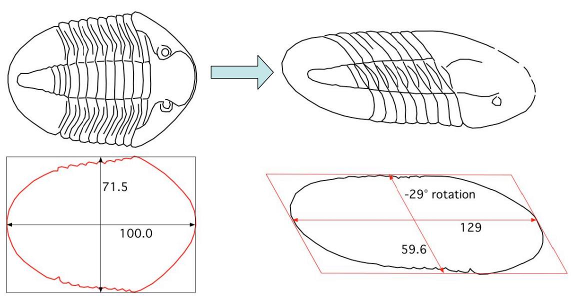
Practical determination in the field

- Example of result for many measurements
- Draw best-fit ellipse through data
  - $e_1 = 2.4$
  - $e_2 = 0.4$
  - $R_f = 6$

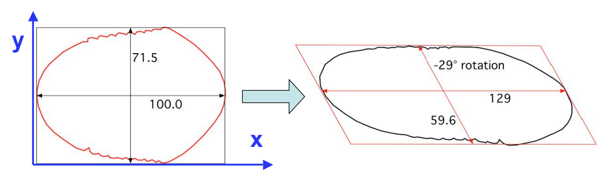


- So, what can you say about the kinematics of deformation?

- The above *Asaphus* trilobite is deformed
- What is the finite strain ratio?
  - We need to know the original shape

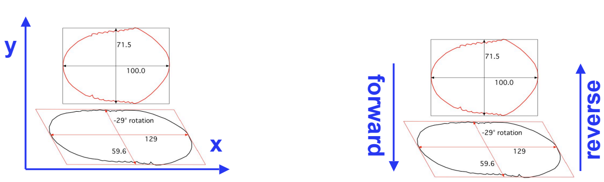


- We know that a normal *Asaphus* has a length-width ratio of 100 / 71.5
- The pleura are at 90° to the long axis of the trilobite
- What is the finite strain ratio?



- Let us define the x-axis // to the axis of the trilobite
- Current length is 129 mm
  - Assume original length 100 mm
  - e_{axis} = 129/100 = **1.29**
- Current width // pleura is 59.6 mm
  - Original width 71.5 mm
  - e_{pleura} = 59.6/71.5 = **0.83**
- Pleura rotated β_{pleura} = **-29°** (anti-clockwise)

Draw the Mohr circle
For strain
Assuming that the x-axis did not rotate, what is the position gradient tensor?



- Draw the Mohr circle for strain
- Assuming our original size assumption was correct, what is the area change?
- Assuming that the *x*-axis did not rotate, what is the position gradient tensor?
- Now assume there was NO area change
  - What is the position gradient tensor (**F**)?
  - What was the original size of the trilobite?
- Finally, draw the Mohr circle for reverse deformation
  - From the deformed state to the undeformed state
  - What is the position gradient for reverse strain ($\mathbf{F}^{-1}$)